

The Most Probable Size of the Universe

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ABSTRACT

It has recently been suggested, by Firouzjahi, Sarangi, and Tye, that string-motivated modifications of the Hartle-Hawking wave function predict that our Universe came into existence from “nothing” with a de Sitter-like spacetime geometry and a spacetime curvature similar to that of “low-scale” models of Inflation. This means, however, that the Universe was quite large at birth. It would be preferable for the initial scale to be close to the string scale [or perhaps the Planck scale]. The problem with this, however, is to explain how any initial homogeneity is preserved during the pre-inflationary era, so that Inflation can indeed begin. Here we modify a suggestion due to Linde and assume that the Universe was born with the topology of a torus; however, we propose that the size of the torus is to be predicted by the FST wave function. The latter does predict an initial size for the torus at about the string scale, and it also predicts a pre-inflationary spacetime geometry such that chaotic mixing preserves any initial homogeneity until Inflation can begin at a relatively low scale.

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1. Creating a String Gas Universe From Nothing

One of the most appealing ideas regarding the origin of our world is that it came into existence out of “nothing” [1][2][3]². Work on this idea has recently been resumed, stimulated by developments in string theory [7][8][9][10][11][12]. In particular, Firouzjahi et al [7][12] take up the question of using the wave function of the Universe to show that the most probable state of the early Universe is one that leads to Inflation. Known models of string Inflation [see, for example, [13]] imply that the inflationary scale should be several orders of magnitude below the string scale. In fact, Firouzjahi et al find that their modified wave function [which takes into account the effects of *decoherence*] predicts that the Universe should come into existence, from “nothing”, at about this inflationary scale.

Because “creation from nothing” tends to be associated with global de Sitter space-time, this means in practice that one shows that the Universe is created in the form of a [perturbed] global dS_4 , with a curvature [proportional to $1/L_{\text{inf}}^2$] which corresponds to the inflationary energy scale. Now in dS_4 itself, L_{inf} plays two roles: it sets the scale *both* for the temporal evolution *and* for the size of the spatial sections [which are spheres, or finite quotients of spheres [14][15][16], of maximal curvature $1/L_{\text{inf}}^2$]. This corresponds to the fact that L_{inf} appears *twice* in the formula for the [Lorentzian, signature $(+ - - -)$] Global de Sitter metric:

$$g(\text{GdS}_4)(L_{\text{inf}})_{+---} = d\tau^2 - L_{\text{inf}}^2 \cosh^2(\tau/L_{\text{inf}}) [d\chi^2 + \sin^2(\chi) \{d\theta^2 + \sin^2(\theta) d\phi^2\}]. \quad (1)$$

In the context of “creation from nothing” this means that, if we model the process using de Sitter spacetime, then [topological complications aside] starting Inflation at a low energy scale forces the Universe to be born [at $\tau = 0$] *with a relatively large size*. It also implies that the shape of the de Sitter Penrose diagram is fixed: it is square if the spatial sections are simply connected, rectangular [and precisely twice as high as wide] if the sections have RP^3 topology [14][15][16], and so on.

However, within string theory it seems rather unnatural for the Universe to be born so large: a size near to the string scale would seem more appropriate. This is particularly true if we subscribe to any version of *string gas cosmology* [17], where the string length scale plays a basic role though T-duality.

Furthermore, as Linde [18][19] has recently emphasised, Inflation necessarily begins with the Universe in a spatially homogeneous state: *but it is not easy to understand why the Universe should be created in such a state if it was born large*. Understanding this question is fundamental for the approach of Firouzjahi et al, which is based on the assumption that a cosmological model can only be acceptable if it passes through an inflationary phase.

These issues are closely related to the deep questions connected with the apparently extremely “special” initial conditions of our Universe, as discussed for example in [20][21]. We certainly will not try to resolve those problems here, but, as a first step, we note that they look more tractable if the Universe was born small — say, at around the string length scale. For we can then hope that specifically “stringy” effects will allow us to explain how the special initial state was selected. But how can the Universe be born with a *spatial* size much smaller than its inflationary *spacetime* scale, L_{inf} ?

²Clear reviews of quantum gravity, including many subtleties which will not be discussed here, are given in [4][5]; see also [6] for an interesting recent development.

The solution suggested by Linde [see also [22]] makes use of the proposal of Zel'dovich and Starobinsky [23], who considered the possibility that the Universe began [perhaps as a Planck-scale fluctuation] with *compact, flat* spatial sections instead of locally spherical ones — that is, the spatial sections have the topology of a three-dimensional torus or of one of the non-singular quotients of a three-torus [24][25]. We can explain the relevance of this idea in the following way: in such cosmologies, *the spatial scale is decoupled from the spacetime curvature scale*. That is, if the spatial tori³ are characterized by a length K , this K can determine the initial size without having any necessary relationship with the spacetime curvature scale L . For an explicit example, take the version of de Sitter spacetime with a spacetime curvature length scale L and a spatial section at time $t = 0$ which is a cubic torus of side length $2\pi K$. The metric of this Spatially Toral de Sitter spacetime is

$$g(\text{STdS})(K, L)_{+---} = dt^2 - K^2 e^{(2t/L)} [d\theta_1^2 + d\theta_2^2 + d\theta_3^2], \quad (2)$$

where the torus is parametrized by angles which run from $-\pi$ to $+\pi$. [This is the metric used by Ooguri et al [9] in their recent study of the duality between black holes and accelerating cosmologies.] Clearly K and L are *entirely independent*, and this will continue to be the case for metrics of this kind on $\mathbb{R} \times T^3$ with more general scale functions.

Linde's suggestion is that the Universe began, at the beginning of a *pre-inflationary phase*, with flat, compact spatial sections of length scale K , and with a spacetime curvature scale L_{inf} . However, the fact that K is independent of L opens up the possibility that K is significantly smaller than L_{inf} . We wish to consider the possibility that K should approximate the string scale. Linde argues, by contrast, that the pre-inflationary era begins at about the *Planck* scale. There are certain advantages to that proposal: Linde stresses that in this way we can take full advantage of the decoupling of spatial and spacetime scales — with flat, compact sections there is no need for any tunnelling at all, so that the creation of such a spacetime is not suppressed. While this proposal is as simple as possible, we believe that starting at the string scale is also worth investigating; for, as mentioned earlier, it is natural from the point of view of string gas cosmology [17], in which the Universe is effectively never smaller than roughly the string scale. The great virtue of this approach is that it allows us to give a stringy resolution of the initial singularity. As we shall review later, singularities are otherwise very hard to avoid in toral cosmologies, so this is a considerable advantage.

In either case, whether the initial scale is Planckian or stringy, the Universe expands from the initial state until it reaches a size comparable to L_{inf} , at which point Inflation is supposed to begin.

The problem with this idea is that, even if we were to succeed in solving the deep problem of “specialness” at the string [or Planck] scale, we need somehow to *maintain* homogeneity until the Universe has expanded to the inflationary scale, since otherwise Inflation will not be able to begin. However, Linde's proposal can automatically take care of this problem, *provided that the pre-inflationary spacetime geometry has a suitable form*. For if the geometry is such that all parts of the Universe remain in causal contact during

³For simplicity, we take the spatial sections to be cubic tori. Everything we shall say applies equally to tori of other shapes and to quotients of tori; the reader merely has to interpret “length scale” in the appropriate way.

the pre-inflationary phase, then any initial homogeneity can then be preserved by causal processes [“chaotic mixing” [26]] until this *global causal contact* is lost.

We thus obtain a very appealing picture: the Universe is born, either from a Planckian fluctuation or from “nothing”, in a very homogeneous state, and with nearly [or even *exactly*] flat, compact spatial sections. The spacetime curvature scale is already L_{inf} , but the spatial scale has a *significantly smaller* value, K . The Universe then expands until the inflationary scale is reached, at which point Inflation will begin if all spatial gradients are [still] sufficiently small by then. But this last condition can be met if the pre-inflationary spacetime geometry allows global causal contact until that time. We refer to this picture as “Linde’s programme”, though we remind the reader that Linde argues for a value of K different from the one to be discussed here.

However, the basic questions now are these: what fixes the value of K relative to L_{inf} ? How is the correct pre-inflationary spacetime geometry chosen? Only if we can answer these questions can we claim to have implemented Linde’s programme for low-scale Inflation.

If we insist on a certain mild energy condition, to be discussed below, then a FRW cosmology with flat spatial sections cannot be “created from nothing”, and we are forced back to the Planck length for K , as Linde advocates. The drawback here is that the origin of the Universe, and the way in which the initial singularity is resolved, can only be described in terms of Planck-scale physics of which we still have little or no understanding.

Here we shall explore what seems to be the only alternative: *we apply the FST wave function to the case where the Universe tunnels from “nothing” to an initial state with compact but flat, instead of locally spherical, spatial sections*. We assume that the inflationary scale is fixed by string theory [13] and focus on determining the most probable value of K , and the most probable spacetime geometry in the earliest Universe. The hope, of course, is to realize the scenario described above.

It should be clear that this is an ambitious undertaking. It is not obvious that merely changing the topology of the spatial sections will have the desired strong effect on the initial scale, changing it from the inflationary scale to the string scale. Likewise, it is far from obvious that, even if we achieve this, the pre-inflationary spacetime geometry will be such as to maintain chaotic mixing long enough to allow Inflation to start. We shall find, however, that the FST wave function *does predict precisely these things*.

Although it is clear that de Sitter spacetime itself will not allow us to implement Linde’s programme, it is still possible that we might be able to use a sufficiently deformed version of dS_4 to do so. We begin with a discussion of this possibility and its difficulties; this discussion allows us to clarify, in terms of the Penrose diagram, the detailed technical requirements which we shall have to satisfy in order for Linde’s programme to work. We conclude that it is simpler to try toral sections. We then briefly summarize the singularity theory of cosmological models with toral spatial sections, emphasising that they can only be created from “nothing” if a certain geometric condition, the *Null Ricci Condition*, is violated. In the next section we use this fact to guide us in constructing explicit examples of simple cosmological models with toral sections which can be created from “nothing”. These form a single-parameter family. We then compute the FST wave function for this family and use it to predict the most probable values of both the family parameter — so that we know which geometry we are [most probably] dealing with — and, with reasonable

assumptions regarding the string length scale, of K/L_{inf} . Though the model is of course over-simplified, the result is quite satisfactory, in the sense that K/L_{inf} is predicted to be small — putting K at about the string scale — and the predicted spacetime geometry does lead to global causal contact which is lost only at about the time Inflation begins. Furthermore, there is reason to hope that more realistic versions will continue to yield satisfactory results.

2. Locally Spherical Spatial Sections

Before turning to the case of flat, compact spatial sections, let us first discuss the possibility of using locally spherical spatial sections to realize Linde’s scenario. Note that locally spherical spatial sections are the standard choice in discussions of creation from “nothing” [7][12], and, as we shall see, there is a sense in which this is the most conservative choice; so we should explain why we shall *not* proceed in this way in this work. Doing so will allow us to explain the technical requirements for Linde’s proposal in this more familiar context.

Simply connected de Sitter spacetime with [in the signature we use here] spacetime curvature $-1/L^2$ is defined as the locus, in five-dimensional Minkowski spacetime [signature $(+ - - - -)$], defined by the equation

$$+ A^2 - W^2 - Z^2 - Y^2 - X^2 = -L^2. \quad (3)$$

This locus has topology $\mathbb{R} \times S^3$, and it can therefore be parametrized by *global* conformal coordinates $(\eta, \chi, \theta, \phi)$ defined by

$$\begin{aligned} A &= L \cot(\eta) \\ W &= L \operatorname{cosec}(\eta) \cos(\chi) \\ Z &= L \operatorname{cosec}(\eta) \sin(\chi) \cos(\theta) \\ Y &= L \operatorname{cosec}(\eta) \sin(\chi) \sin(\theta) \sin(\phi) \\ X &= L \operatorname{cosec}(\eta) \sin(\chi) \sin(\theta) \cos(\phi). \end{aligned} \quad (4)$$

Here χ, θ, ϕ are the usual coordinates on the three-sphere, and η is angular conformal time, which takes its values in the interval $(0, \pi)$. [Conformal time itself is then given by ηL .] The metric of Global de Sitter spacetime is then

$$g(\text{GdS}_4)(L)_{+---} = L^2 \operatorname{cosec}^2(\eta) [d\eta^2 - d\chi^2 - \sin^2(\chi) \{d\theta^2 + \sin^2(\theta) d\phi^2\}]. \quad (5)$$

An obvious conformal transformation allows us to extend the range of η , so that it takes all values in $[0, \pi]$. The Penrose diagram is clearly square in the case of *simply connected* spatial sections, since χ also has this range in that case. [It will be narrower in the non-simply-connected case, since, for example, χ only runs to $\pi/2$ on \mathbb{RP}^3 .]

Let us briefly recall how to use this diagram to explain how Inflation solves the horizon problem. In Figure 1, the triangular structures above the upper horizontal line [which schematically represents both the end of Inflation and decoupling] give the usual representation of the horizon problem: events observed in opposite directions apparently have no common past. Below the line, deeper into de Sitter spacetime, however, we see that

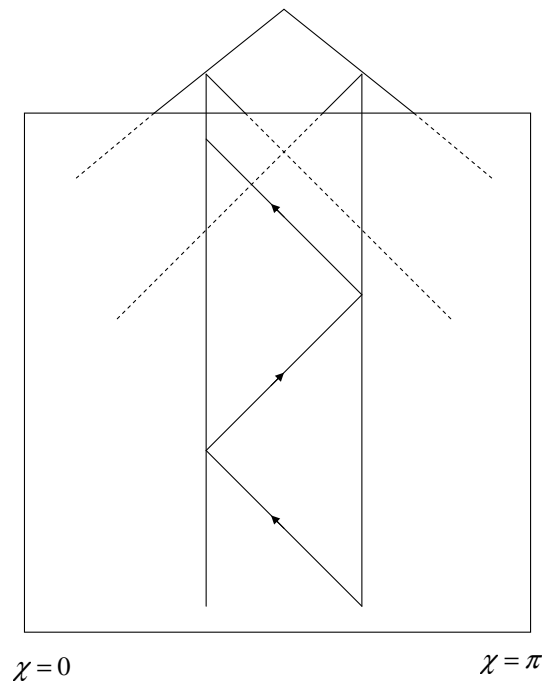


Figure 1: How Inflation Solves the Horizon Problem.

the two events do in fact share much of their past. Furthermore, inside the shared region there has been ample opportunity for objects with worldlines passing through these events to exchange signals. This explains the similarity of the conditions observed at the two events in question. Notice that the explanation has a definite “holographic” flavour: the behaviour on the de Sitter boundary is understood in terms of interactions in the bulk. Notice too that the topology of the spatial sections is irrelevant to this argument: it would not matter, for example, if the Penrose diagram were rectangular rather than square, as would be the case if the spatial sections were assumed to be copies of $\mathbb{R}P^3$. [Of course, the size of the post-inflationary part of the diagram has been enormously exaggerated in Figure 1.]

This explanation works because the events we see in opposite directions are, by inflationary standards, not very remote from each other. The explanation would be considerably less convincing if we were discussing objects at $\chi = 0$ and $\chi = \pi$. For, even asymptotically, these objects share only half of their past events in the simply connected

case, and there is *no* opportunity for signals to be exchanged even in that region. Actually, the situation here is even worse than it appears. For global de Sitter spacetime has both a contracting phase and an expanding phase. We usually delete the former, on the grounds that it will be cut off either by a singularity or simply in the course of constructing the usual Hartle-Hawking Euclidean-to-Lorentzian transition. In this case we should retain only the upper half of the Penrose diagram. In the simply connected case, this would mean that antipodal observers have *no* past events in common, and they are completely out of causal contact. Clearly, simply connected de Sitter spacetime would not allow us to explain homogeneity on a full spacelike slice, if such an explanation were required.

The point of this discussion is that if we are hoping to use a pre-inflationary era to prepare suitable conditions for the beginning of Inflation, as Linde proposes [18][19], and if we insist on using a spacetime with locally spherical spatial sections, then the spacetime geometry will have to be very different from that of de Sitter spacetime.

During the pre-inflationary era, however, the inflaton energy does not yet completely eclipse other forms of energy. We have not yet taken into account the effect of these other forms of energy on the spacetime geometry. Now a theorem of Gao and Wald [27] implies that if a spacetime is globally hyperbolic [with compact sections] and null geodesically complete, and if it satisfies the Null Energy Condition [NEC] and the null generic condition⁴, then any observer will be able to “see” an entire spacelike slice at some sufficiently late time — which is *not* possible in exact, simply connected de Sitter spacetime. Essentially what happens in such a spacetime is that null geodesics sent out from any point are able to “turn around” and begin to return towards that point. [One says that the spacetime has *no null lines* [28]: on every inextensible null geodesic there exists a pair of points with timelike separation.] This means that the introduction of any kind of generic matter satisfying the NEC will cause the simply connected de Sitter Penrose diagram to become taller than it is wide, so that null geodesics can “bounce” off one or both of the vertical edges of the diagram, as discussed in [29][30][31].

The Gao-Wald theorem only describes the *ratio* of the height to the width of the perturbed de Sitter Penrose diagram; it does not tell us how the actual minimal size of a spatial section is affected. In fact, however, the effect of introducing matter which satisfies the NEC is to make the Universe *smaller* at its minimum size: that is, if the spatial sections are spheres, the minimum possible radius is L in de Sitter space, but it is less than L in the perturbed spacetime. This is related to the tendency of such matter to focus null geodesics. In the case in which the matter is introduced in such a way as to preserve the FRW character of the spacetime, this can be seen directly from the Friedmann equation, which, with Distorted de Sitter metric

$$g(\text{DdS})(L)_{+---} = d\tau^2 - L^2 a^2(\tau/L) [d\chi^2 + \sin^2(\chi) \{d\theta^2 + \sin^2(\theta) d\phi^2\}] \quad (6)$$

takes the form

$$L^2 \dot{a}^2 = \frac{8\pi L_P^2}{3} \rho L^2 a^2 - 1, \quad (7)$$

⁴This is the requirement that, along every null geodesic, there should exist a point where the tangent vector k^a and the curvature R_{abcd} satisfy $k_{[a} R_{b]cd[e} k_{f]} k^c k^d \neq 0$. Exact de Sitter spacetime itself does *not* satisfy this condition, which is why the Gao-Wald conclusion is not true of it.

where $a(\tau)$ is the dimensionless scale function⁵, L_P is the Planck length, L is the length scale in the initial de Sitter spacetime, and ρ is the total energy density [de Sitter energy density plus the matter we are introducing into de Sitter spacetime]. Our objective is to deform de Sitter spacetime, but not to the extent of causing it to become singular — that would rule out creation from “nothing”. Therefore we assume that $a(\tau)$ still has a minimum value, a_{\min} . From (7) we have

$$\frac{8\pi L_P^2}{3} \rho(a_{\min}) L^2 a_{\min}^2 - 1 = 0. \quad (8)$$

Writing $\rho = 3/(8\pi L_P^2 L^2) + \rho_*$, where $3/(8\pi L_P^2 L^2)$ is the energy density of de Sitter spacetime and where ρ_* is the energy density we are introducing, we have now

$$1 - a_{\min}^2 = \frac{8\pi L_P^2}{3} \rho_*(a_{\min}) L^2 a_{\min}^2; \quad (9)$$

clearly the minimum value of the scale factor is smaller than its value in de Sitter spacetime, which of course is unity. Thus the minimum radius, $a_{\min}L$, is indeed smaller than the de Sitter value, L .

The metric now is still conformal to the standard metric [see the bracketed expression on the right side of equation (5)] on the product of a closed interval with a local three-sphere, so the Penrose diagram remains rectangular and of the same width; but, as we explained above, the Gao-Wald theorem implies that the height of the diagram must increase. Hence the total angular conformal time [starting from the minimal radius section, since we are not interested in a contracting phase] always exceeds $\pi/2$.

Thus we see that the effect of taking into account the back-reaction of the matter content is to make the de Sitter minimal radius smaller and to make the Penrose diagram taller. Since the density of non-inflaton matter in the pre-inflationary phase is relatively large [compared to the inflationary era, when it is completely dominated by the inflaton energy], this effect can be very important.

It should be clear from our discussion above that small initial radii and tall Penrose diagrams are precisely what we require in order to carry out Linde’s programme for low-scale Inflation. For of course the “tallness” will allow particles and signals to be sent back and forth between all parts of the Universe. The era during which this is possible is what we call the “pre-inflationary” era. Causal contact begins to be lost when the cosmological horizon begins to develop. Since Inflation can only begin if the matter distribution is sufficiently homogeneous, *it is essential that the Universe should have reached the Inflationary scale by this time.* We shall refer to the subsequent phase as the inflationary era. This is the top square in Figure 2; that is, the inflationary era must begin roughly π units of angular conformal time below the upper boundary of the Penrose diagram [which is at angular conformal time $\eta = \Omega$].

Thus Figure 2 portrays Linde’s programme: the pre-inflationary era must be such that homogeneity is maintained until the surface $\eta = \Omega - \pi$ is reached, which is roughly when Inflation itself should begin; the reader can picture this line as a compact hypersurface of typical “inflationary size”. As a general summary of Linde’s programme, this diagram

⁵Note that, with these conventions, the minimal value of the scale function is *not* equal to unity, except in the case of pure de Sitter spacetime.

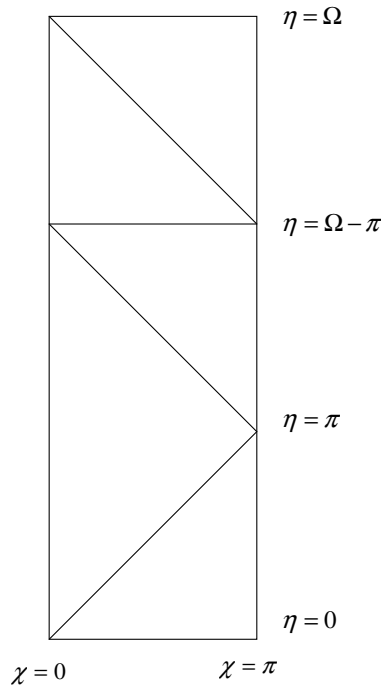


Figure 2: Penrose diagram for the pre-inflationary/inflationary Eras.

[with suitable changes of coordinate labels] is valid for any spatial topology, provided that the sections are *compact*. In particular, this same diagram gives the conditions for the programme to work in the case of *toral* spatial sections⁶.

The question is now: how small can the initial radius be, and how tall can the Penrose diagram become, in the case of locally spherical spatial sections?

Physically, one might reason as follows. Clearly, there must be a bound, imposed by self-consistency, on how much [conventional] matter can be born with a Universe which is created from “nothing”. For if there is too much matter initially, it will dominate the inflaton to such an extent that the Strong Energy Condition [SEC] will be satisfied *initially*, even if it is violated later when the conventional matter dilutes. The classical singularity theorems [32], applied to this early phase alone, will require the spacetime to be singular — which of course would mean that the Universe was *not* born from “nothing”.

⁶ χ will in that case be replaced by one of the angular coordinates on the torus, taken to run from $-\pi$ to $+\pi$.

Given that the introduction of conventional matter causes the minimum radius to shrink and the total conformal time to increase, one would expect to be able to arrange to have an arbitrarily small initial radius, and an arbitrarily tall Penrose diagram, by carefully adjusting the amount of matter to be sufficiently close to the critical value. *For locally spherical sections, only the second part of this statement is correct, however.*

To see this, let us consider a concrete example. To be specific, we shall assume that the Universe is born with locally spherical structure, with a positive cosmological constant having energy density ρ_Λ , and containing some kind of radiation with initial density ρ_{0R} . As always in discussions of creation from “nothing”, we assume that the Euclidean-to-Lorentzian transition takes place along a surface of zero extrinsic curvature, that is, a surface of minimal size: so ρ_{0R} is also the *maximal* radiation density. The assumption that the matter takes the form of radiation is motivated by [12], where indeed it is found that the modified wave function of the Universe naturally leads to the creation of exactly this kind of spacetime. We shall find it useful to define a parameter

$$\kappa = \rho_{0R}/\rho_\Lambda. \quad (10)$$

Now because radiation satisfies the SEC, there is an upper bound on κ ; otherwise, as explained above, the combination of the cosmological constant and the radiation will also satisfy the SEC initially, and then the classical singularity theorems [applied to the earliest phase, before the radiation energy density drops to the point where the SEC is violated] will imply the presence of an initial singularity. The SEC is violated initially [and therefore at all times] provided that

$$[\rho_{0R} + \rho_\Lambda] + 3[\frac{1}{3}\rho_{0R} - \rho_\Lambda] < 0, \quad (11)$$

where we have used the respective equations of state for the cosmological constant and for radiation⁷. This simply means that we must have

$$\kappa < 1 \quad (12)$$

if we are to deform de Sitter spacetime without causing a singularity.

Assuming now that we are dealing with a FRW model described by equation (7), we can write the radiation density as

$$\rho_R(\tau) = \frac{3\alpha}{8\pi L_P^2 L^2} a^{-4}, \quad (13)$$

where α is a positive constant. Since the energy density associated with the cosmological constant is $3/8\pi L_P^2 L^2$, we have, from equation (7),

$$L^2 \dot{a}^2 = \alpha a^{-2} + a^2 - 1. \quad (14)$$

Thinking of the right side as an effective potential, and noting again that we are interested in spacetimes which are perturbations of de Sitter spacetime [in the sense that, like the latter, they have a minimum value of the scale function rather than a maximum], one sees

⁷Actually, the spacetime remains non-singular even if this inequality is saturated. But then the extrinsic curvature of a spatial slice is never zero — it only approaches zero asymptotically to the past.

that the minimum value of $a(\tau)$ is found by taking the larger positive root of $a^4 - a^2 + \alpha$; that is,

$$a_{\min} = \sqrt{\frac{1}{2}[1 + \sqrt{1 - 4\alpha}]}. \quad (15)$$

Notice that there is an upper bound on α ; this will turn out to be equivalent to (12), that is, it just expresses the requirement that the SEC should be violated at all times.

Since the maximum density occurs when $a(\tau)$ is smallest, substituting (15) into (13) allows us to express the maximal radiation density, which we denoted ρ_{0R} above, in terms of α . Solving for α , we can express it in terms of ρ_{0R} and thus in terms of κ :

$$\alpha = \frac{\kappa}{[1 + \kappa]^2}. \quad (16)$$

Substituting this into (15) we have an expression for the minimum value of the scale function in terms of κ :

$$a_{\min} = \frac{1}{\sqrt{1 + \kappa}}. \quad (17)$$

Notice that, as expected, the minimum value of $a(\tau)$ is always less than its de Sitter counterpart, unity.

Provided that (12) is satisfied, the Penrose diagram in the simply connected case will be a rectangle of width π ; the height will be given by the full extent of [angular] conformal time, beginning at the start of the Lorentzian era; we denote this height by Ω_κ . This is easily computed by solving (7) for $d\eta = dt/La(\tau)$ and integrating:

$$\Omega_\kappa = \int_{a_{\min}}^{\infty} \frac{da}{\sqrt{a^4 - a^2 + \alpha}}, \quad (18)$$

where α is given by (16) and a_{\min} is given by (17). It is important to bear in mind that indeed a_{\min} does depend on κ ; otherwise (18) would give the impression that the introduction of matter [positive α] *shortens* the Penrose diagram, which, as we know, is never the case for matter with a positive density. Notice finally that Ω_κ is the angular conformal time expended during the *expansion* of this spacetime: the contracting phase is cut away, since it is to be replaced by a Euclidean space in the construction of the wave function. Thus in particular $\Omega_0 = \pi/2$, as in the *expanding* phase of simply connected de Sitter, *not* π .

We are now in a position to answer our questions. First, we see at once from (12) and (17) that, unfortunately, it is *not* possible to reduce the initial size of the Universe very substantially by assuming that it is born containing radiation. Even in the limiting case, in which κ approaches unity, the reduction of the radius is only to approximately 71% of its de Sitter value. If the latter has its usual inflationary value, then the Universe is *not* born at the string scale [which is usually estimated to be about two orders of magnitude smaller than this], no matter how much radiation is born with it.

On the other hand, if we let $\kappa \rightarrow 1$ in (18), the effect will be to cause the two roots of the quadratic in a^2 to coincide, causing a logarithmic divergence; however, the integral always converges for any value of κ strictly less than unity. Thus, Ω_κ can be made *arbitrarily large* by choosing κ sufficiently close to unity. Hence we can make the Penrose diagram as tall as we wish. Unfortunately, however, large values of Ω_κ can be achieved only at

the cost of a fairly severe fine-tuning, because in fact Ω_κ grows extremely slowly as κ approaches unity. Thus we have to think carefully about the extent of vertical stretching we need to achieve in order to achieve chaotic mixing, as in Linde’s programme.

Recalling the way we used the Penrose diagram [Figure 1] to explain the workings of conventional Inflation, we must at least require that the pre-inflationary era should belong to the shared past of antipodal observers, and that it should be possible for these observers to exchange at least one signal during that era. [In reality, of course, we would want more communication than this, but we are seeking here to establish a lower bound.] For that, we must stretch the “expanding” [upper] half of the simply connected de Sitter Penrose diagram by a factor of at least 6 — see Figure 2 — so that its height becomes 3π . This allows for precisely one complete circumnavigation during the pre-inflationary era. But a numerical investigation of the integral in (18) shows that, even to achieve this most conservative case, one needs κ to be at least 0.99997. Combining this with (12), we have

$$0.99997 < \kappa < 1. \quad (19)$$

If one wishes to ensure that it should be possible for signals and objects to perform multiple circumnavigations in this era, then of course the fine-tuning becomes substantially worse.

In short, it is not easy to realise Linde’s pre-inflationary scenario using locally spherical spatial sections. This is *not* to say that it cannot be done. For example, one could try to obtain smaller initial radii by considering Universes born with other kinds of matter apart from radiation. This can in fact be made to work. Furthermore, the Penrose diagram can be made substantially narrower by considering topologically non-trivial versions of de Sitter spacetime, such as the one, mentioned earlier, with $\mathbb{R}P^3$ spatial sections. There is of course no physical justification whatever for assuming that global de Sitter spacetime should be simply connected; on the contrary, there are very good reasons for supposing that it isn’t [15][33]. For global de Sitter spacetime [with locally spherical sections] this means that the topology of the underlying manifold is taken to be $\mathbb{R} \times [S^3/\Gamma]$, where Γ is a finite group from a well-known list [34].

Apart from $\mathbb{R}P^3$, the manifolds S^3/Γ are not globally isotropic, and most of them are not homogeneous, so, strictly speaking, one cannot draw a Penrose diagram for the corresponding versions of global de Sitter spacetime; nevertheless we can argue as follows. First, in view of our objectives here, it is reasonable to confine attention to the homogeneous quotients of S^3 ; furthermore, we are not interested in spaces where the quotient is smaller than S^3 in some directions but not in others [as is the case for most of the lens spaces]. The obvious candidate is the well-known binary icosahedral quotient S^3/I_{120}^* , where I_{120}^* is the group, of order 120, which is the double cover of the group of symmetries of a regular icosahedron. For a given curvature, S^3/I_{120}^* is up to ten times smaller than S^3 ; that is, it is ten times smaller in certain directions, but the reduction is less dramatic in other directions. As the reduction in volume is clearly by a factor of 120, we can use five as an estimate of the reduction in the width of the “Penrose diagram” for this version of de Sitter spacetime. Thus the diagram is somewhat taller than wide, even before we take into account the presence of matter.

Combining all these observations, we can put together a picture of the kind proposed by Linde, while using locally spherical spatial sections. That is, we can construct a pre-inflationary Universe which begins on a small spatial section and which is represented

by a Penrose diagram which is somewhat taller than it is wide: this allows all parts of a spatial section to remain in causal contact until Inflation is ready to begin. What we have learned, however, is that, firstly, this prescription is not unique [there are many kinds of matter which could be born with the Universe, and there are many possible spatial topologies], and, secondly, the argument only works with special choices from this range of possibilities. This in itself is not a drawback: the whole “wave function of the Universe” philosophy is based on the idea that the wave function can make such choices for us. The problem lies in trying to imagine *how* this selection can be made. How can the wave function select the appropriate matter which appears when the Universe is born? Still more difficult: how can the wave function express its preference for [say] the binary icosahedral group as the fundamental group of the spatial sections? Notice that the usual S^4 instanton cannot be used for this purpose [35]; one would need a new instanton, possibly with an orbifold structure⁸. Of course, one can argue that questions of this kind will ultimately have to be confronted by *any* theory of cosmic origins; but the point is that such questions must be answered immediately if we are to use locally spherical sections in Linde’s programme.

Although we do not regard these arguments as a conclusive demonstration that locally spherical sections cannot be used here, it does seem appropriate to try a simpler alternative. As was explained in the Introduction, the size of the spatial sections is decoupled from the spacetime curvature scale if we use flat, compact spatial sections, so we now turn to this simpler alternative.

3. Global Structure of Spatially Flat Accelerating Cosmologies

In this section we shall assume that our Universe was born with *flat* spatial sections, since we have seen that this is actually a simpler procedure than using locally spherical sections.

In fact, of course, proceeding in this way brings us into agreement with standard practice in astrophysics. Note that some recent observations favour spatial flatness even more strongly than the well-known WMAP data, at the one percent level [$\Omega_k = -0.010 \pm 0.009$ [37]; see however the cautionary notes in [38]]. While such data cannot of course rule out locally spherical spatial sections if these are sufficiently large, and while it may be true that some of the classical justifications for the assumption of spatial flatness are questionable [39][40], there are still good theoretical reasons [25] to prefer flat sections, *provided* however that these are compact. The simplest argument is that string theory abounds with extended objects such as strings and branes: so it is natural to consider spaces, like tori and toral quotients, in which one can place these objects in such a way that they either cannot contract to a point [17], or can do so only with great difficulty [41]. Note in this connection that one of the more promising approaches to string phenomenology is based on the assumption that the “small” dimensions correspond to spaces which are [singular] quotients of tori [42]. It is then extremely natural to assume that the “large” dimensions also take the form of a [non-singular] quotient of a torus. There are in fact

⁸However, lest the reader gain the impression that fundamental groups of locally spherical spaces cannot be selected in a *physical* way, let us note that in fact there are concrete suggestions as to how it can be done: see [36].

many other theoretical reasons to prefer flat, compact spatial sections, and we shall discuss some of them in the Conclusion.

Again, it is actually standard practice in the Inflation literature to describe de Sitter spacetime itself as having flat spatial sections. This “Cartesian” version of the de Sitter metric makes use of Cartesian coordinates (t, x, y, z) , running from $-\infty$ to $+\infty$, on \mathbb{R}^4 . The metric of “Cartesian de Sitter” is

$$g(\text{CdS})(L)_{+---} = dt^2 - e^{(2t/L)} [dx^2 + dy^2 + dz^2]. \quad (20)$$

How can this metric on \mathbb{R}^4 be equivalent to the metric given above on $\mathbb{R} \times S^3$? The short answer is that it can’t: this metric turns \mathbb{R}^4 into a spacetime which is timelike and null geodesically incomplete. To see this, note that the Cartesian coordinates (t, x, y, z) are related to the ambient Minkowski coordinates by

$$\begin{aligned} A &= -L \sinh(t/L) - \frac{1}{2L} (x^2 + y^2 + z^2) e^{t/L} \\ W &= +L \cosh(t/L) - \frac{1}{2L} (x^2 + y^2 + z^2) e^{t/L} \\ Z &= z e^{t/L} \\ Y &= y e^{t/L} \\ X &= x e^{t/L}. \end{aligned} \quad (21)$$

Now in (21) we have

$$W - A = L e^{t/L}, \quad (22)$$

so $W > A$ everywhere in the domain of the coordinates. There is no such constraint on the global conformal coordinates, and indeed the region $W > A$ corresponds, by the first two equations in (4), to the region $\eta > \chi$ of the Penrose diagram, the “upper left-hand” triangular half. That is, “Cartesian de Sitter” is obtained by simply cutting away half of the full de Sitter spacetime. Despite appearances, then, *this spacetime is geodesically incomplete* [43][44].

The incompleteness of “spatially flat de Sitter” takes on quite a different aspect if we switch to the toral spatial topology $\mathbb{R} \times T^3$. To see precisely how this works, take the Cartesian coordinates and consider the hypersurface $t = 0$. By equations (4) and (22), the surface $t = 0$ corresponds to the line in the Penrose diagram given by

$$\cos(\eta) + \sin(\eta) = \cos(\chi), \quad (23)$$

as shown in Figure 3.

Now take the two-sphere of radius πK in this surface; it is represented by the heavy dot in Figure 3. This two-sphere can be enclosed in a cube of side length $2\pi K$. We are really interested in this cube, but we shall discuss it in terms of the enclosed sphere. We allow the sphere to evolve along with the expansion: that is, we let its points trace out timelike geodesics perpendicular to $t = 0$. By the last three equations in (4) and the last three in (21), we can represent this by the curve

$$L \operatorname{cosec}(\eta) \sin(\chi) = \pi K e^{(t/L)} = \pi K [\operatorname{cosec}(\eta) \cos(\chi) - \cot(\eta)], \quad (24)$$

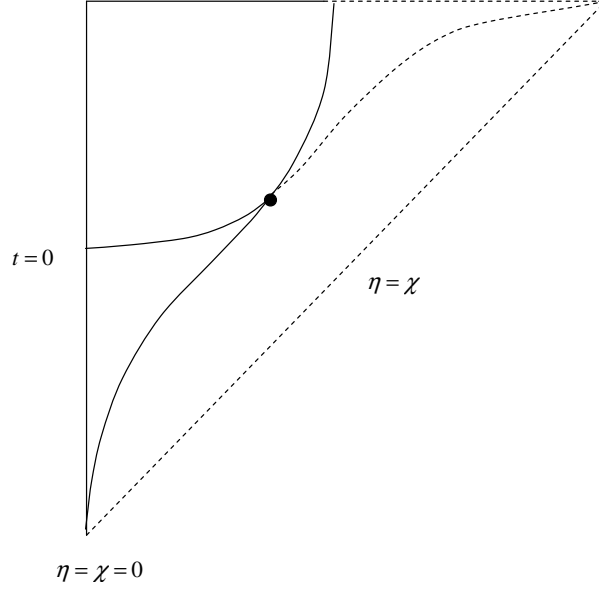


Figure 3: Penrose Diagram of Toral de Sitter [solid lines].

or

$$\cos(\eta) = \cos(\chi) - \frac{L}{\pi K} \sin(\chi). \quad (25)$$

This is also shown in Figure 3. If we now compactify the Cartesian spatial section $t = 0$ to a cubic torus of side length $2\pi K$, and allow this torus to follow the geodesic shown [in the sense that it always just encloses the two-sphere which had radius πK at $t = 0$], then we have constructed spatially toral de Sitter spacetime. Putting $x = K\theta_1$, $y = K\theta_2$, $z = K\theta_3$ [where we recall that the angles $\theta_{1,2,3}$ run from $-\pi$ to $+\pi$, so that the initial distance from the centre of the cube to the boundary planes is πK], we obtain the metric given in equation (2). To obtain the Penrose diagram, we just have to discard all that part of Figure 3 [indicated by dotted lines] which lies to the right of the timelike geodesic

shown⁹. But that means that no part of our new spacetime comes near to the line $\eta = \chi$ — *except* near the single point $(\eta = 0, \chi = 0)$.

Contrary to statements often found in the literature, however, the toral version of de Sitter spacetime is, like the version with non-compact spatial sections, *geodesically incomplete*. The only difference is that the incompleteness is now associated with a single point $(\eta = 0, \chi = 0)$. As this point is in the infinite past according to the observers corresponding to the time coordinate t in equation (2), it is understandable that the incompleteness here has not been widely noticed. The fact that toral de Sitter spacetime is incomplete nevertheless follows from powerful singularity theorems for asymptotically de Sitter spacetimes proved by Andersson and Galloway [45][46]. The point is that while the timelike geodesics associated with t are complete, *other timelike [and null] geodesics are not*. If we trace back the worldline of a freely falling object, then we find that the total proper time can be either infinite or finite, depending on the way the worldline winds around the torus. Since the torus is shrinking as we move backwards in time, the finite proper time case is easy to achieve; in fact, that is the generic situation for timelike geodesics, as is discussed in [46].

We conclude, then, that switching from locally spherical to flat [compact or non-compact] spatial sections has a strong effect on de Sitter spacetime: it forces it to become [null and timelike] geodesically incomplete. The toral case *does* represent a great improvement over the non-compact case, but only in the sense that objects can only enter the spacetime through a single point, and not through the entire region $\eta = \chi$. But, on the other hand, the absence of a “true” [curvature] singularity here is an artifact of the very special spacetime geometry — *generically*, the incompleteness at $(\eta = 0, \chi = 0)$ *will* turn into a curvature singularity. In other words, the toral case is more similar to a conventional cosmological model than the non-compact case. Like a conventional cosmological model, however, it will generically have an initial curvature singularity.

This is of course not very welcome from the point of view of creation from “nothing”. In fact, the situation even in the case of compact sections is much worse than it seems, for geodesic incompleteness is an extremely general phenomenon for spacetimes of this kind. Indeed, one finds that the details of what is happening in the incomplete region are almost entirely irrelevant: the Andersson-Galloway theorems imply that, as long as a spacetime with $\mathbb{R} \times T^3$ topology is *asymptotically* de Sitter [and non-singular at late times], then, essentially¹⁰, it *has* to be incomplete to the past as long as the *Null Ricci Condition* [see below] continues to hold. In short, we have another example of the phenomenon which is such a remarkable feature of the classical singularity theorems: the incompleteness is *not* a result of assumptions about symmetries, the special [FRW] form of the metric, or even about the details of the Einstein equation.

The only way to avoid the conclusions of the Andersson-Galloway theorems is to violate the Null Ricci Condition [NRC], which requires that the Ricci tensor should satisfy

$$R_{\mu\nu} k^\mu k^\nu \geq 0. \quad (26)$$

for every *null* vector k^μ . This condition is obviously satisfied by de Sitter spacetime, which

⁹Of course, this is an approximation, valid to the extent that we are able to represent the cube by the enclosed sphere.

¹⁰“Asymptotically de Sitter and non-singular at late times” means, more precisely, that the spacetime has a *regular future spacelike conformal boundary* and is *future asymptotically simple*.

is why the version with toral sections has to be incomplete. As it stands, the NRC is a purely geometric condition, but, if we assume that the Einstein equations hold *exactly*, then it is equivalent to the *Null Energy Condition* [NEC], which is just the requirement that the energy-momentum tensor $T^{\mu\nu}$ should satisfy

$$T_{\mu\nu} k^\mu k^\nu \geq 0 \quad (27)$$

for all null k^μ . If, as seems at first reasonable from this point of view, one insists that the NRC should hold, then an asymptotically de Sitter spacetime with toral spatial sections must be singular to the past if it is non-singular in the far future. Typically, the Penrose diagram will either be “triangular” — as in Figure 3, but with the point of incompleteness generically replaced by a genuine, curvature singularity — or rectangular, as in Figure 2, but with a curvature singularity at the bottom of the diagram.

In short, generic toral cosmologies *are inevitably singular if the NRC holds*. This means that the spacetime in the Zel’dovich-Starobinsky [23] theory *will* in reality [that is, if it contains any matter] be singular; it also, of course, means that a toral universe cannot be created from “nothing” if the NRC holds.

From the point of view of string theory, and specifically of string gas cosmology [17], this is a very puzzling conclusion. For in that approach, the use of toral spatial sections is precisely what allows the theory to *avoid* an initial singularity, through T-duality. We conclude [31] that string gas cosmology demands a violation of the NRC in the very earliest Universe — that is, precisely around the region of spacetime we are discussing here, the pre-inflationary era — if we follow Linde in assuming that the spatial sections of our Universe are flat, compact three-manifolds. All this is of course in sharp contrast to the case of locally spherical sections. [The Andersson-Galloway results do *not* demand singularities in that case even if the NRC holds; the technical reason being that the fundamental groups of the sections are always *finite* in that case.]

What we have learned in this section is this: if the Universe is to be created from “nothing” in the form of a flat three-space, then the Universe must be born containing some structure which violates the NRC. This structure will modify the shape of the Penrose diagram in Figure 3 and, as we shall see, can make it resemble Figure 2. This will of course strongly affect the computation of the wave function of the Universe.

In order to proceed, we need some explicit examples of such NRC-violating spacetimes. We now present a family of such examples.

4. NRC Violation in the Early Universe: Explicit Examples

Our objective in this section is to discuss a simple family of spacetimes describing the creation, from “nothing”, of a spacelike torus. Later we shall also show how, and to what extent, stringy considerations constrain its shape, but in this section our discussion is essentially classical. We shall be guided by the conclusions of the previous section.

The precise observational and theoretical [47] status of the NEC [defined above] has been much debated in recent years. In particular, it is well known that violations of the NEC can be dangerous: the result is often a future singularity [29][48]. If such behaviour cannot be avoided, then of course there can be no hope of using a spacetime geometry

with toral spatial sections as a model of the pre-inflationary Universe. Fortunately, that is not the case, as we shall soon see.

The first point to note is that, in contexts like the present one, there is no reason to believe that the Einstein equation should hold exactly, and so violations of the NRC and of the NEC *are two very different matters* here. It has been specifically noted in the case of braneworld models [49][50][51] that there are explicit corrections to the Einstein equation which allow the NRC to be violated *while every matter field satisfies the NEC* [52]. Similarly, the NEC and the NRC can be significantly different in certain Gauss-Bonnet and other variants of Einstein gravity [53][54]. Therefore we should not assume that violating the NRC will necessarily lead to all of the well-known complications which may [or may not] arise when the NEC is violated.

In order to discuss this situation concretely, we shall write the equation governing the metric in the Einstein form, that is, as

$$G^{\mu\nu} = 8\pi L_P^2 [T_{\text{Matter}}^{\mu\nu} + T_{\text{NRC}}^{\mu\nu}], \quad (28)$$

where $T_{\text{NRC}}^{\mu\nu}$ represents the NRC-violating effects we have just been discussing; in other words, if k^μ is null, $8\pi L_P^2 T_{\text{NRC}}^{\mu\nu} k_\mu k_\nu$ just quantifies the failure of $8\pi L_P^2 T_{\text{Matter}}^{\mu\nu} k_\mu k_\nu$ to agree with $G^{\mu\nu} k_\mu k_\nu$ [$= R^{\mu\nu} k_\mu k_\nu$]. We shall refer to the appropriate components of this tensor, ρ_{NRC} and p_{NRC} , as “energy” density and “pressure”, while bearing in mind that these terms should not be taken literally. In particular, these components can have unconventional signs.

In a pre-inflationary regime, one assumes that, initially, some kind of matter is present which postpones the dominance of the inflaton energy. There are two simple ways in which this can happen. The first is the conventional kind of behaviour, discussed in Section 2 above, where the density of the matter decreases with the expansion, eventually leaving the [essentially constant] energy of the inflaton to dominate. There is a second possibility, however: the additional “matter” [corresponding to $T_{\text{NRC}}^{\mu\nu}$] could have a negative “energy” density. If [the absolute value of] *this* density decays to zero with the expansion, the effect will be that the total energy *increases* towards its inflationary value, instead of decreasing towards it; but the final effect will be the same. However, if the total energy density *increases* with the expansion, then the Hubble parameter will also increase: and this implies that the NRC must be violated.

Our discussion of the Andersson-Galloway theorems above implies that something of this sort *must* happen if a torus is to be created from “nothing”. This result is extremely general and does not depend on assuming an FRW form for the metric. However, if we do make that assumption, then we can see in detail why the NRC has to be violated here. In this case the metric has the form

$$g(K)_{+---} = dt^2 - K^2 [a(\frac{t}{L_{\text{inf}}})]^2 [d\theta_1^2 + d\theta_2^2 + d\theta_3^2], \quad (29)$$

in an obvious generalization of equation (2); the Friedmann equation in this case is

$$\dot{a}^2 = \frac{8\pi L_P^2}{3} \rho a^2. \quad (30)$$

For the Universe to be created from “nothing”, there must be a surface of vanishing extrinsic curvature. The Friedmann equation immediately demands that the total density

has to vanish everywhere along this initial slice. Thus any positive energy which may be present [such as that of the inflaton] must initially be exactly cancelled by some *negative* “energy” density, given by evaluating ρ_{NRC} along the initial surface. If $|\rho_{\text{NRC}}|$ now decays with the expansion, the result, as explained above, will necessarily be a violation of the NRC.

In view of all this, we shall assume that the Universe is born, from “nothing”, containing [a] the inflaton, [b] conventional matter satisfying the NEC, and [c] the NRC-violating component just discussed. Our objective now is to construct *explicit* examples spacetimes with the above properties: that is, we wish to find spacetimes which, in violating the NRC, remain non-singular and have a surface with the topology of a torus and with zero extrinsic curvature. This surface should replace the point in Figure 3 where geodesic completeness fails, and — we hope — lead to a Penrose diagram like the one shown in Figure 2.

For our purposes here, it is essential to obtain an exact solution for the spacetime metric — the wave function of the Universe can be computed only if we are able to evaluate the Euclidean version of the action, and we shall need an explicit expression for the volume form, and so on. This will of course require somewhat drastic simplifications. The first simplification is that we shall assume that the inflaton can be represented by a positive cosmological constant with energy density $3/(8\pi L_P^2 L_{\text{inf}}^2)$ and pressure $-3/(8\pi L_P^2 L_{\text{inf}}^2)$. The second is that we shall simply ignore the contribution of the conventional matter. This is justified by the now-familiar fact that such matter only tends to stretch the Penrose diagram vertically if it does not cause a singularity. Including it, therefore, would only improve our results. Finally, we shall greatly simplify our calculations by assuming that the “pressure” corresponding to the negative “energy density” ρ_{NRC} is given by

$$p_{\text{NRC}} = w_{\text{NRC}} \rho_{\text{NRC}}, \quad (31)$$

where the “equation-of-state parameter” w_{NRC} is a *constant*, at least approximately; we believe that this approximation, a standard one in astrophysics, does not affect any of our conclusions.

With these three assumptions, we can now proceed in the usual way. The vanishing of the covariant divergence of the total stress-energy-momentum tensor leads to the conclusion that ρ_{NRC} is proportional to $a^{-3(1+w_{\text{NRC}})}$. Since we want $|\rho_{\text{NRC}}|$ to decay with the expansion, we must have $w_{\text{NRC}} > -1$. Since the inflaton energy density $3/(8\pi L_P^2 L_{\text{inf}}^2)$ cancels the pressure $-3/(8\pi L_P^2 L_{\text{inf}}^2)$, we have

$$\rho + p = \rho_{\text{NRC}} + p_{\text{NRC}} = \rho_{\text{NRC}} [1 + w_{\text{NRC}}], \quad (32)$$

and then $w_{\text{NRC}} > -1$ implies that the NRC is violated here, since ρ_{NRC} is *negative*.

It is convenient to define a constant γ by

$$\gamma = 3[1 + w_{\text{NRC}}]; \quad (33)$$

from the above discussion, γ can take any *positive* value. The presence of this new parameter is a direct result of our need to violate the NRC: in fact, γ measures the maximum extent to which the NRC is violated [see below]. As we are about to see, however, the spacetime geometry of the pre-inflationary era is now entirely fixed, apart from this parameter. Thus, γ also parametrizes our ignorance of the spacetime geometry

of the earliest Universe. It will in fact turn out that the ability of the FST wave function to fix γ will be crucial in realizing our scenario.

We see that ρ_{NRC} is some negative multiple of $a^{-\gamma}$. If we define K in equation (29) such that the cubic torus has an initial side length $2\pi K$, so that the initial value of $a(t)$ is unity, then the fact that $3/(8\pi L_P^2 L_{\text{inf}}^2)$ must be cancelled initially allows us to fix the constant of proportionality between ρ_{NRC} and $a^{-\gamma}$: evidently we must have

$$\rho_{\text{NRC}} = \frac{-3}{8\pi L_P^2 L_{\text{inf}}^2 a^\gamma}. \quad (34)$$

Adding this to the inflaton energy, substituting the total into equation (30) and solving, we obtain a family of metrics parametrized by γ :

$$g(\gamma, K)_{+---} = dt^2 - K^2 \cosh^{(4/\gamma)}\left(\frac{\gamma t}{2L_{\text{inf}}}\right) [d\theta_1^2 + d\theta_2^2 + d\theta_3^2]. \quad (35)$$

Clearly these metrics are entirely non-singular, despite the fact that the NRC is violated everywhere: there is *no* “Big Rip”. [In fact, it was precisely for this reason that these metrics were first introduced [29].] Furthermore, they do all that we ask: the extrinsic curvature is zero at $t = 0$, where a torus of side length $2\pi K$ is created from “nothing”; the initial size of the torus will be small compared to the inflationary scale as long as K/L_{inf} is small. The Universe then expands, and, since $\cosh^{(4/\gamma)}(\frac{\gamma t}{2L_{\text{inf}}}) \rightarrow e^{(2t/L_{\text{inf}})}/2^{(4/\gamma)}$, the metric eventually approaches the inflationary form¹¹. In fact, for the values of γ of interest to us here, the approach to the inflationary metric is very rapid as measured by proper time. It proves to be much slower, however, in *conformal* time, and this is of course vital if we are to obtain a Penrose diagram of the desired shape.

The precise geometric role of γ may be explained as follows: with a natural choice of null vector k^μ we find that the Ricci tensor of the metric (35) satisfies

$$R_{\mu\nu} k^\mu k^\nu = \frac{-\gamma}{L_{\text{inf}}^2} \text{sech}^2\left(\frac{\gamma t}{2L_{\text{inf}}}\right). \quad (36)$$

Setting $t = 0$, we see that γ measures the initial [and maximal] extent of NRC violation. For other fixed t , however, this expression tends to zero as γ becomes large, yet its integral from 0 to ∞ is $-2/L_{\text{inf}}$, independent of γ ; thus the effect of taking γ to be large is to focus the NRC violating effect close to $t = 0$, while small values of γ describe a more “diffuse” form of NRC violation.

One of these metrics, namely $g(2, K)_{+---}$, was recently discussed [55] in connection with an attempt to resolve the Big Bang singularity in spatially flat FRW cosmologies, using the higher-derivative corrections of the Einstein-Hilbert Lagrangian implied by string theory. This can be done while avoiding ghosts, again underlining our view that violations of the NRC need not be interpreted in terms of actual physical fields having negative energy densities. It should be noted that all of the technical virtues [55] of $g(2, K)_{+---}$ [connected with the behaviour of ΔR , the spacetime Laplacian of the scalar curvature] are actually shared by $g(\gamma, K)_{+---}$ for *all* γ . This is fortunate, since, as was discussed in [31] [see below], $g(2, K)_{+---}$ itself is not acceptable in string theory.

¹¹See equation (2); K must be adjusted by a factor of $2^{2/\gamma}$.

We can write equation (35) in the form

$$g(\gamma, K)_{+---} = K^2 \cosh^{(4/\gamma)}\left(\frac{\gamma t}{2L_{\text{inf}}}\right) [d\eta^2 - [d\theta_1^2 + d\theta_2^2 + d\theta_3^2]], \quad (37)$$

where $\cosh^{(4/\gamma)}(\frac{\gamma t}{2L_{\text{inf}}})$ is to be understood as a certain function of angular conformal time, η . If we take one axis of the torus, say in the direction of θ_1 , to be the spacelike axis, then we have a “directed Penrose diagram” which is a rectangle of width π and of height

$$\Omega(\gamma, K) = \frac{2L_{\text{inf}}}{\gamma K} \int_0^\infty \frac{dx}{\cosh^{(2/\gamma)}(x)}. \quad (38)$$

As in Figure 2, the pre-inflationary era lasts from $\eta = 0$ until $\eta = \Omega(\gamma, K) - \pi$. During this era, the scale factor increases from unity to $\cosh^{(2/\gamma)}(X)$, where X is the solution of the equation

$$\pi = \frac{2L_{\text{inf}}}{\gamma K} \int_X^\infty \frac{dx}{\cosh^{(2/\gamma)}(x)}. \quad (39)$$

Now our objective here, as explained in Section 2 above, is to ensure that K is small compared to the inflationary length scale, and that $\Omega(\gamma, K)$ is at least 3π — preferably much larger, so that there is ample [conformal] time in which antipodal observers can exchange signals. In fact, $\Omega(\gamma, K)$ is a decreasing function of both K and γ , and so to achieve our objectives we must understand how *both* of these parameters are fixed, not just K .

This is an essential point. It is *not* true that a small value of K/L_{inf} automatically ensures that the Penrose diagram will be tall: for that, we need also to ensure that the terms involving γ in (38) are not too small. In other words, the pre-inflationary spacetime geometry must also be selected with care. Even if K indeed proves to be small, *this is of little use to us if γ is such that chaotic mixing fails before the inflationary scale has been reached.*

In fact, γ will appear throughout our discussion, and it is essential to understand how to constrain it. We propose that it should be determined in much the same way as K : that is, the wave function of the Universe should [help to] select the most probable value of γ . The rather daunting task before us is to show *both* that K is approximately equal to the string scale *and* that γ is such that chaotic mixing fails only when the Universe has expanded to the inflationary scale.

The relevant Euclidean versions of our family of metrics are given simply by complexifying time in (35), yielding

$$g(\gamma, K)_{----} = -dt^2 - K^2 \cos^{(4/\gamma)}\left(\frac{\gamma t}{2L_{\text{inf}}}\right) [d\theta_1^2 + d\theta_2^2 + d\theta_3^2]; \quad (40)$$

of course, $(----)$ signature is equivalent to $(++++)$. Here Euclidean time is defined on $(-\pi L_{\text{inf}}/\gamma, 0]$; at $t = 0$ there is the usual transition from Euclidean to Lorentzian signature. The Euclidean space is singular at $t = -\pi L_{\text{inf}}/\gamma$; we shall comment on this later.

Thus we have a concrete example of a family of early-Universe spacetime geometries which *may* permit the effects Linde requires for Inflation to begin. Furthermore, all that is required of the wave function is that it should select two numerical parameters, K and

γ ; we do not need to select the detailed topology or to know more details of the “matter” content. This is of course a great simplification compared to the case of locally spherical spatial sections.

We shall now proceed to investigate how acceptable values of K and γ might be derivable from the wave function of the Universe.

5. The Hartle-Hawking Wave Function In The Toral Case

In this section we begin to explore the possibility that the basic parameters of our model are selected by the cosmic wave function.

In the recent work [for example, [7][9][12]] on creating the Universe from “nothing”, it is assumed that the Hartle-Hawking wave function, or a modification of it, can be used to describe such a process. We remind the reader that this assumption has not been accepted by all [56]; Linde argues that the HH wave function describes a ground state, but *not* its creation. On the other hand, the work of Ooguri et al [9] appears to provide evidence in favour of the “creation interpretation”, within the framework of topological string theory. Our view is that, ultimately, a proper interpretation [or extension] of the wave function may allow these points of view to be reconciled. In the meantime, the most pressing need is to see whether the “creation interpretation” can actually be made to work. To that end, we shall provisionally assume that the HH and FST wave functions do predict the probability of the creation of a Universe with specified properties. We also assume, without further discussion, the validity of the usual assumptions: that the “mini-superspace” approach is valid and that semi-classical saddle-point approximations can be used. These standard assumptions and their limitations are discussed in detail in [4][5].

According to [7][12], the Hartle-Hawking wave function describes the [non-normalized] probability of tunnelling to de Sitter spacetime, with spacetime curvature scale L , from “nothing”. Up to constant factors this probability is given by

$$P_{HH} = \exp(\pi L^2/L_P^2), \quad (41)$$

where again L_P is the Planck length. Clearly this favours arbitrarily large values of L . This is not acceptable physically, and so the Hartle-Hawking wave function must be modified. The relevant modifications will be discussed in the next section. For the present we shall retain the Hartle-Hawking wavefunction and explain the [rather non-trivial] procedure required to adapt it to tunnelling to a spacetime with toral spatial sections of initial *non-zero* side length $2\pi K$. For concreteness we shall assume that the spacetime geometry is given by some member of the family of metrics given by (35). Bear in mind that the spacetime curvature length scale has a fixed value L_{inf} throughout this discussion.

We saw in the preceding Section that the relevant Euclidean version is given by (40): negative values of t correspond to the Euclidean regime, while positive values of t describe the Lorentzian geometry, and the two are to be conjoined at $t = 0$. Now the Hartle-Hawking instanton with spherical spatial sections is non-singular, even at the point where the scale factor vanishes, because it is [part of] a four-sphere. However, this is highly non-generic behaviour: normally one must expect the vanishing of the scale factor to produce a singularity. In fact, the vanishing of the Euclidean scale factor is itself no bad

thing: indeed, if the scale function did not vanish, there would be a wormhole through to another Euclidean region, which in turn would have further wormholes. In general this would cause the volume of the Euclidean region, and therefore, in general, the action, to diverge. This forces us to perform a manual cutoff at a “plate”, as in [12], which is not entirely satisfactory.

In any case, the Euclidean instanton for a cosmology with toral spatial sections *has to be singular*. One can see this in the following somewhat roundabout way. We know that the NRC has to be violated here: this means that the sum of the total energy density and the total pressure has to be negative. By Einstein’s equation, this means that the Hubble parameter H has to increase: the time derivative is

$$\dot{H}(t) = -4\pi L_P^2(\rho + p), \quad (42)$$

which is positive here. But the left side of this equation involves two time derivatives of the scale factor. Upon complexification, therefore, we will find that the Euclidean version of the right side must become *negative*; for example, in the case of the metric (40) one has

$$\dot{H}(g(\gamma, K)_{----}) = -\frac{\gamma}{2L_{\text{inf}}^2} \sec^2\left(\frac{\gamma t}{2L_{\text{inf}}}\right). \quad (43)$$

Therefore the Euclidean scale factor behaves exactly like the scale factor of a FRW cosmology, with toral spatial sections, which *satisfies* the NRC. But the Andersson-Galloway results tell us that such a cosmology has to be singular, and so the scale factor of the original Euclidean space must vanish at some finite t . This point *is* indeed singular, because [unlike in the case of S^4] the intrinsic curvature of each section is zero here. In the case of (40), the singularity is at $t = -\pi L_{\text{inf}}/\gamma$.

We do not regard such a singularity as a drawback in itself; as we have seen, the vanishing of the scale factor can help to keep the action finite by cutting off the volume. On the other hand, a singularity can itself cause the action to diverge. Following Hawking and Turok [57], we shall not rule out a singular instanton *provided that it does not cause the action to diverge*. Let us investigate this point.

First let us see what happens if we simply compute the original Hartle-Hawking wave function for the NRC-violating geometry given by equation (35). In the signature we are using here, the Euclidean action of the instanton corresponding to (40) has the general form

$$S_E = \frac{1}{16\pi L_P^2} \int R \sqrt{g} dt d\theta_1 d\theta_2 d\theta_3 + S_{\text{NRC}}, \quad (44)$$

where R is the scalar curvature and S_{NRC} is the “action” corresponding to the NRC-violating “matter” discussed earlier. In a *purely formal* way one can obtain an explicit expression for this “action” by constructing a “phantom scalar” [47] model which artificially mimics the effects of the terms involving ρ_{NRC} and p_{NRC} in the Einstein equation. This is non-trivial, because we must take care to do this in a way which is consistent with the assumption that w_{NRC} is *constant*. It can however be done: if one considers a formal field ψ with a reversed kinetic term, and uses an “axion-like” potential, it is possible to show that the result is an “energy density” and “pressure” related by a constant which can be expressed as in equation (33). [The details may be found in [31].] The corresponding Lagrangian proves to be a constant multiple of $\sec^2(\frac{\gamma t}{2L_{\text{inf}}})$, and so by computing the

determinant of the metric given in (40) we find the action S_{NRC} :

$$S_{\text{NRC}} = \left(\frac{\gamma}{3} - 1\right) \frac{3}{8\pi L_{\text{P}}^2 L_{\text{inf}}^2} \int_{-\pi L_{\text{inf}}/\gamma}^0 \sec^2\left(\frac{\gamma t}{2L_{\text{inf}}}\right) [8\pi^3 K^3 \cos^{(6/\gamma)}\left(\frac{\gamma t}{2L_{\text{inf}}}\right)] dt. \quad (45)$$

We stress that the “field” ψ is merely a device for arriving at this result: it plays no further role.

The scalar curvature in this signature is given by

$$R(g(\gamma, K)_{----}) = -\frac{12}{L_{\text{inf}}^2} + \frac{3}{L_{\text{inf}}^2} (4 - \gamma) \sec^2\left(\frac{\gamma t}{2L_{\text{inf}}}\right), \quad (46)$$

so we can compute its contribution to the action as in (45). Combining this with (45), we find after simplifications that the total Euclidean action becomes

$$S_{\text{E}}(\gamma, K) = \frac{6\pi^2 K^3}{L_{\text{inf}} L_{\text{P}}^2 \gamma} \int_{-\pi/2}^0 \left\{ \left(1 - \frac{\gamma}{6}\right) \sec^2(x) - 2 \right\} \cos^{(6/\gamma)}(x) dx. \quad (47)$$

The first point to make regarding this integral is that it diverges for all values of $\gamma > 6$. *This is the result of the fact that the instanton is singular.* The integral converges for all values less than or equal to 6, but there is discontinuous behaviour at $\gamma = 6$. We have in fact

$$\begin{aligned} S_{\text{E}}(\gamma, K) &= \text{Undefined}, \quad \gamma > 6 \\ S_{\text{E}}(6, K) &= \frac{-2\pi^2 K^3}{L_{\text{inf}} L_{\text{P}}^2} \\ S_{\text{E}}(\gamma, K) &= \frac{-6\pi^2 K^3}{L_{\text{inf}} L_{\text{P}}^2 \gamma} \Delta_{(6/\gamma)}, \quad \gamma < 6, \end{aligned} \quad (48)$$

where, for any non-negative constant β , Δ_{β} is defined as

$$\Delta_{\beta} = \int_0^{\pi/2} \sin^{\beta}(x) dx. \quad (49)$$

Notice that since $\Delta_1 = 1$, $|S_{\text{E}}(6, K)|$ is *twice as large* as the limiting value of $|S_{\text{E}}(\gamma, K)|$ as γ approaches 6 from below.

Leaving aside inessential factors, the Hartle-Hawking probability of creating a Universe with toral sections and parameters γ, K is

$$P_{\text{HH}}(\gamma, K) = e^{-S_{\text{E}}(\gamma, K)}. \quad (50)$$

Now one can show that $\Delta_{(6/\gamma)}/\gamma$ is a decreasing function of γ , *unbounded above* when γ is sufficiently small. From (48) we therefore see that the tunnelling probability can be made arbitrarily large either by increasing K or by decreasing γ . Various interpretations are possible here. One is that the most probable way for the Universe to be born is with arbitrarily large K , taking us back to the \mathbb{R}^3 version of de Sitter spacetime [equation (20)]. This would leave γ undetermined. Unfortunately it is equally valid to interpret the result to mean that γ should vanish, while leaving K undetermined, which of course does not make sense geometrically. In any case it is clear that the wave function here is not normalisable and so its interpretation is obscure.

Unsurprisingly — we have not taken into account decoherence [7][12] effects — the result of our calculation is not satisfactory. Nevertheless we have learned something important: the parameter γ is now strongly constrained, since the action diverges if γ exceeds 6. *No such constraint exists classically*: γ can take any positive value in the metric (35). The mere existence of the wave function imposes this constraint. The point is that the Euclidean instanton is, as we saw, necessarily singular in the toral case; and while this singularity has the virtue of automatically cutting off the volume of the instanton, the singular geometry itself will cause the action to diverge unless γ is constrained. The requirement that the spatial topology should be toral, combined with the demand that the wave function must be well-defined if the Universe is to be born at all, strongly constrains the initial geometry. As a matter of general principle, this is a very desirable situation: we would hope that considerations of internal consistency should strongly constrain or even determine the initial geometry, since this must otherwise remain hard to explain. Thus it is a hopeful sign that the possible range of γ is indeed greatly reduced by such arguments. On the other hand, the fact remains that the Hartle-Hawking wave function seems to favour arbitrarily small values of γ , which does not make sense. We shall have to return to this point after considering the Firouzzjahi-Sarangi-Tye wave function in this case.

6. The FST Wave Function In The Toral Case

Sarangi and Tye [12] stress that the impossibility of normalizing the Hartle-Hawking wave function is in itself evidence that the wave function has to be modified. Ideally, the requisite modification here should be derived from string theory. String theory does have some very general properties which are surely relevant, and we can therefore attempt to guess the general form the modifications must take. For example, in string theory spacetime is ultimately 10-dimensional, and there is a natural length scale, the string scale L_s . These general facts give us some guidance as to the form of a modified wave function. On the other hand, it is stressed by Firouzzjahi, Sarangi, and Tye [7][12] that the wave function must also be shaped by quantum decoherence induced by an “environment” consisting of perturbative modes of the spacetime geometry, together with whatever matter is created in the beginning.

Combining these basic observations, we expect a correction to the Hartle-Hawking wave function which depends on the 10-dimensional volume of the Euclidean instanton, measured in string units. Firouzzjahi et al begin with a spacetime with locally spherical spatial sections and spacetime length scale L ; the specific form they propose for the non-normalized probability is

$$P_{\text{FST}} = \exp\left(\frac{\pi L^2}{L_P^2} - c \frac{V_{10}}{L_s^{10}}\right), \quad (51)$$

where L_s is the string length scale, V_{10} is the ten-dimensional volume corresponding to the Euclidean instanton describing the tunnelling of the Universe from “nothing”, and c is a constant which is to be calculated [from quantum decoherence theory] given the precise details of the vacuum. Since V_{10} depends on L , the presence of two competing factors in the expression for P_{FST} immediately means that arbitrarily large values of L

may no longer be preferred, and indeed Firouzjahi et al are able to argue, beginning with de Sitter spacetime with spherical sections, that the “KKLMMT” inflationary scenario [58] is the most probable way for the Universe to be born. Our objective here is to see what the modified wave function predicts if we assume that the Universe is born, instead, with *toral* spatial sections. For concreteness we shall assume that the spacetime geometry is described by some member of the family of metrics given by (35), leaving the parameter γ undetermined for the moment. As usual, we fix L at L_{inf} henceforth, since the initial size of the Universe is determined by K , not L .

The four-dimensional volume of the instanton is easily computed:

$$V_4 = 8\pi^3 K^3 \int_{-\pi L_{\text{inf}}/\gamma}^0 \cos^{(6/\gamma)}\left(\frac{\gamma t}{2L_{\text{inf}}}\right) dt = \frac{16\pi^3 K^3 L_{\text{inf}}}{\gamma} \Delta_{(6/\gamma)}. \quad (52)$$

It is very natural to assume here that the internal space is a toral orbifold, as for example in [42]. In our three-torus, $2\pi K$ can be defined as the length of the shortest closed geodesic: this is the most natural definition, in view of the importance of *circumnavigations* in our whole approach. Now clearly we have to find an explicit formulation of the intuitive idea that the initial size of all dimensions should be the “same”, as in string gas cosmology [17]. In the case of the orbifold, it is not possible to insist on this literally, but we can argue that the *average* size of the internal dimensions should be the same as the initial size of the three dimensions which are destined to become large: that is, it should be about $2\pi K$. As this is the length of the shortest closed geodesic on the three-torus, and since — again as in string gas cosmology — T-duality leads us to expect a minimum effective size for all dimensions, one might argue that we should say that the size of the internal dimensions should be *at least* $2\pi K$, but let us be conservative and use this value. Then the internal volume is well approximated by $[2\pi K]^6$. Thus we have

$$V_{10} = \frac{[2\pi]^9 K^9 L_{\text{inf}}}{\gamma} \Delta_{(6/\gamma)}, \quad (53)$$

and so the FST wave function in our case yields a probability function

$$P_{\text{FST}}(\gamma, K) = \exp\left\{-S_E(\gamma, K) - c \frac{[2\pi]^9 K^9 L_{\text{inf}}}{\gamma L_s^{10}} \Delta_{(6/\gamma)}\right\}. \quad (54)$$

According to the equations (48) this is

$$\begin{aligned} P_{\text{FST}}(\gamma, K) &= \text{Undefined, } \gamma > 6 \\ P_{\text{FST}}(6, K) &= \exp\left\{\frac{2\pi^2 K^3}{L_{\text{inf}} L_P^2} - c \frac{[2\pi]^9 K^9 L_{\text{inf}}}{6 L_s^{10}}\right\} \\ P_{\text{FST}}(\gamma, K) &= \exp\left\{\left[\frac{6\pi^2 K^3}{L_{\text{inf}} L_P^2} - c \frac{[2\pi]^9 K^9 L_{\text{inf}}}{L_s^{10}}\right] \frac{\Delta_{(6/\gamma)}}{\gamma}\right\}, \quad \gamma < 6. \end{aligned} \quad (55)$$

We see at once that large values of K are no longer favoured, and this is of course highly desirable. Equally, however, we see that the FST wave function does not in itself resolve the γ problem: for any given value of K , we can make $P_{\text{FST}}(\gamma, K)$ arbitrarily large simply by choosing γ to be sufficiently small. Again, this does not make sense either geometrically [equation (35)] or in terms of having a normalizable wave function.

It is worth stressing this point. One might have thought that generalizing the FST wave function to the toral case would be a straightforward matter: the wave function should favour a particular value of K here, just as it favours a particular value of L in the locally spherical case. This proves to be incorrect. The ultimate reason for this is again the Andersson-Galloway results, which tell us that the toral topology will lead to a singularity unless we violate the NRC. But violating the NRC forces us to introduce a new parameter, γ , which itself strongly affects the structure of the wave function. One can certainly expect difficulties like this to arise in general, not just for the particularly simple family of metrics we have used here.

As we have emphasised, classically there is no constraint on γ other than that it should be positive: any value makes sense in equation (35). But we saw that internal consistency — the requirement that the wave function should be well-defined — imposes a strong constraint on γ when the quantum theory is constructed. This constraint is the *upper* bound $\gamma \leq 6$. Our only hope now is that similar self-consistency arguments can impose a *lower* bound on γ . This is in fact precisely what happens, as we now explain.

7. How Non-Perturbative String Physics Constrains γ

We argued earlier that it is necessary for the NRC to be violated if the Universe is to be created from “nothing” as a torus. However, we also argued that this could be done without violating the Null Energy Condition [as opposed to the Null *Ricci* Condition]. It might seem, therefore, that violating the NRC may not have any physical consequences provided that we decouple the NRC from the NEC.

In [31] it was pointed out, however, that in string theory there is a known source of *non-perturbative* instability *which is related only to the NRC, not to the NEC*. This is the specifically stringy process discovered by Seiberg and Witten [59]. Take a BPS $(D - 1)$ -brane together with an appropriate antisymmetric tensor field in a manifold which is asymptotic to the $(D + 1)$ -dimensional hyperbolic space H^{D+1} . The brane action takes the form [59]

$$S_B = T(A - \frac{D}{L}V), \quad (56)$$

where T is the tension of the brane, A is its area, V the volume enclosed, and L is the length scale of the asymptotic hyperbolic space. The point is that this action is a purely geometric object. If the geometry of the ambient space is such that [for example] this action is unbounded below, then the result will be a severe instability due to the nucleation of “large branes”, *no matter how* the geometry came to have that particular shape.

In fact, Seiberg and Witten were able to show that this non-perturbative¹² stringy instability is never a problem if the conformal structure at infinity is represented by a metric of positive scalar curvature; on the other hand, it is unavoidable if the scalar curvature at infinity is negative. The case of scalar-flat [including, of course, completely flat] boundaries is particularly delicate: Seiberg-Witten instability occurs in some cases but not in others¹³.

¹²The instability is non-perturbative in the sense that a barrier has to be overcome to create the branes.

¹³A particularly clear discussion of this and of related issues is given by Kleban et al in [60].

In [31], it was found that NRC-violating spacetimes with flat compact spatial sections frequently [not invariably] do suffer from Seiberg-Witten instability: that is, string theory frequently rules out models which appear to be classically consistent. In particular, if we consider the asymptotically hyperbolic version of any spacetime of the form given in equation (29), we find that *the brane action is always unbounded below whenever the NRC is violated and the NRC-violating term decays towards infinity at the same rate as, or more slowly than, $a(t)^{-3}$* . From equations (32) and (34) we see that this just means that values of γ less than or equal to 3 are absolutely forbidden here. This is precisely the lower bound we need.

The situation for values of γ greater than 3 but less than 6 is much more delicate. The action for a Seiberg-Witten “large brane” in the relevant geometry here, $S_B(\gamma, K, t)$, is always positive at $t = 0$, but it then decreases; it is bounded below if $\gamma > 3$, but the limiting value is given by [31]

$$\lim_{t \rightarrow \infty} S_B(\gamma, K, t) = 2^{(3 - \frac{6}{\gamma})} \pi^3 K^3 T \frac{(\gamma - 6)(3 + \gamma)}{\gamma(\gamma - 3)}. \quad (57)$$

We immediately see that the *largest* value permitted by the existence of the wave function, 6, is the *smallest* value which guarantees that the brane action should never be negative. This is very remarkable, and certainly suggests that $\gamma = 6$ is the preferred value. For values strictly between 3 and 6, the brane action does become negative for sufficiently large t , and it continues to decrease thereafter: there is no minimum for any finite t . This means that we can create a brane/antibrane pair and, by moving them to sufficiently large t , we can reduce the action. Admittedly the reduction is only by a finite amount, but, as in the similar cases discussed by Maldacena and Maoz in [61], there is a danger of non-perturbative instability for these values of γ also.

This suggests that we should exclude *all* values of γ below 6, leaving 6 as the only physically acceptable value of γ . However, that may be premature. Notice that the brane action becomes negative only beyond a certain critical value of t , t_c , which depends on γ . If γ is close to 6, then t_c can be very large, and indeed it may occur beyond the end of the pre-inflationary period. But in that region we have no reason to believe that the NRC continues to be violated; indeed, since our assumption is that the NEC is satisfied at all times, and since the Einstein equations are presumably approximately valid during the inflationary era, we must expect that NRC violation ceases at some point. At that point, the brane action may begin to increase, so it may never have become negative. Thus we should be cautious about excluding values of γ near to [but less than] 6.

A detailed investigation shows that this argument still excludes most values of γ up to a value quite close to 6. Rather than pursue these details, however, we shall find it more instructive to allow γ to vary in the range $3 < \gamma \leq 6$. The wave function itself will select the appropriate value of γ in this range.

We can summarize the findings of this section very briefly. Classically, the crucial parameter γ can range between 0 and infinity. The self-consistency of the “creation from nothing” scenario requires γ to be no greater than 6; the self-consistency of a string-theoretic formulation of the relevant spacetime geometry requires γ to be greater than some value below, but quite close to, 6. With these constraints we can fix the most probable value of γ and therefore of K .

8. The Most Probable Values of K and γ

Let us finally compute the most probable values of our two basic parameters, using both the Hartle-Hawking and the FST wave functions.

According to the HH wave function, $|S_E(\gamma, K)|$ measures the probability of tunnelling with given values of K and γ . By the equations (48) we know that the HH wave function favours arbitrarily large values of K. Let us assume a cutoff at some value K_{cut} of K which is very large compared to L_P and L_{inf} [and independent of γ], and ask which value of γ is preferred within the range (3, 6] discussed earlier. Now $\Delta_{(6/\gamma)}/\gamma$ decreases everywhere on (0, 6), but recall the curious fact that the action is *discontinuous* at 6. To clarify the consequences of this, we compare three values or limiting values of $|S_E(\gamma, K_{\text{cut}})|$ to see where the maximum lies. We find that

$$\begin{aligned} \frac{L_{\text{inf}} L_P^2}{2\pi^2 K_{\text{cut}}^3} |S_E(6, K_{\text{cut}})| &= 1 \\ \frac{L_{\text{inf}} L_P^2}{2\pi^2 K_{\text{cut}}^3} |S_E(3, K_{\text{cut}})| &= \pi/4 \\ \frac{L_{\text{inf}} L_P^2}{2\pi^2 K_{\text{cut}}^3} \lim_{\gamma \rightarrow 6} |S_E(\gamma, K_{\text{cut}})| &= 1/2. \end{aligned} \quad (58)$$

Here, as agreed in the previous Section, we have allowed γ to go as low as 3. Evidently $\gamma = 6$ is the favoured value. The *extent* to which it is preferred is measured by the fractions

$$\begin{aligned} P_{\text{HH}}(6, K_{\text{cut}})/P_{\text{HH}}(3, K_{\text{cut}}) &= \exp \left\{ \frac{2\pi^2 K_{\text{cut}}^3}{L_{\text{inf}} L_P^2} \left[1 - \frac{\pi}{4} \right] \right\} \\ P_{\text{HH}}(6, K_{\text{cut}})/\lim_{\gamma \rightarrow 6} P_{\text{HH}}(\gamma, K_{\text{cut}}) &= \exp \left\{ \frac{\pi^2 K_{\text{cut}}^3}{L_{\text{inf}} L_P^2} \right\}. \end{aligned} \quad (59)$$

Since we are assuming that K_{cut} is much larger than L_P and L_{inf} , both numbers on the right here are enormous. Thus the Hartle-Hawking wave function strongly favours $\gamma = 6$, reconfirming the [of course, entirely independent] argument in the previous section, which was based on the self-consistency of the string theory formulation.

Although this calculation was unacceptably vague, in that the HH wave function does not determine K, it does illustrate two points. The first is related to the simple fact that creating the Universe is obviously not an experiment that we can perform more than once. Hence it is vital that all probability distributions in such discussions should be very strongly peaked; otherwise we should not know how to interpret the results. The second point is that *discontinuities in the probability function can be useful*, for they may allow us to make very precise predictions. In the present case, for example, $\gamma = 6$ is enormously more probable than $\gamma = 5.99999$. One should look for other examples of this sort.

Now let us turn to the FST wave function. Here the discontinuity complicates the analysis, since the favoured value of K is different for $\gamma = 6$ and values of γ arbitrarily close to 6. From the equations (55) we see that K^* , the most probable value of K, is either the value K_6^* that maximizes $P_{\text{FST}}(6, K)$ or the value K_3^* that maximizes $P_{\text{FST}}(3, K)$. [Notice that this same value, K_3^* , actually maximizes $P_{\text{FST}}(\gamma, K)$ for each fixed γ strictly less than 6.] One finds

$$\left(\frac{K_6^*}{L_{\text{inf}}} \right)^6 = 2 \left(\frac{K_3^*}{L_{\text{inf}}} \right)^6 = \frac{1}{(2\pi)^7} \frac{1}{c} \left(\frac{L_s}{L_P} \right)^2 \left(\frac{L_s}{L_{\text{inf}}} \right)^8. \quad (60)$$

Substituting (60) back into (55) we can compute ratios analogous to those in (59) above:

$$\begin{aligned}\frac{P_{\text{FST}}(6, K_6^*)}{P_{\text{FST}}(3, K_3^*)} &= \exp \left\{ \frac{1}{3\sqrt{c}(2\pi)^{3/2}} \left[1 - \frac{\pi}{4\sqrt{2}} \right] \left(\frac{L_s}{L_P} \right)^3 \left(\frac{L_s}{L_{\text{inf}}} \right)^2 \right\} \\ \frac{P_{\text{FST}}(6, K_6^*)}{\lim_{\gamma \rightarrow 6} P_{\text{FST}}(\gamma, K_3^*)} &= \exp \left\{ \frac{1}{3\sqrt{c}(2\pi)^{3/2}} \left[1 - \frac{1}{2\sqrt{2}} \right] \left(\frac{L_s}{L_P} \right)^3 \left(\frac{L_s}{L_{\text{inf}}} \right)^2 \right\}.\end{aligned}\quad (61)$$

Obviously, several quantities in these relations are not known exactly; fortunately, however, the results are not very sensitive to the details of our choices. Reasonable estimates are as follows. First, Firouzjahi et al [7] estimate c at about 10^{-3} ; although the value is somewhat model-dependent, we shall follow them and use this value. [We shall see that our prediction for the initial size differs considerably from that of FST; by choosing the same value of c , we can make it clear that this is not due to a different choice of parameters.] Next, we shall assume that the string scale is about two orders of magnitude below the Planck scale and that the inflationary scale is about two orders of magnitude lower again. Before proceeding, we emphasise that these values, or any values not drastically different, cause the functions $P_{\text{FST}}(6, K)$ and $P_{\text{FST}}(3, K)$ to become extremely sharply peaked about their maxima: even a 10% variation in K away from the preferred value causes them to decrease by many orders of magnitude. Thus, once again, although the predictions made by the wave function are probabilistic, they are very robust. This is clear also in the case of γ : with these data, we find

$$\begin{aligned}\frac{P_{\text{FST}}(6, K_6^*)}{P_{\text{FST}}(3, K_3^*)} &\approx 8.4 \times 10^{12} \\ \frac{P_{\text{FST}}(6, K_6^*)}{\lim_{\gamma \rightarrow 6} P_{\text{FST}}(\gamma, K_3^*)} &\approx 6.2 \times 10^{18}.\end{aligned}\quad (62)$$

As before, $\gamma = 6$ is favoured over all other admissible values of γ , including [especially] values close to 6; and evidently the prediction $\gamma = 6$ is sufficiently robust. We can now use $\gamma = 6$ in (60) to compute $K^* = K_6^*$.

We find that K^*/L_{inf} is about $1/270$. Substituting this into the formula (38) for the height of the Penrose diagram, using $\gamma = 6$, we find

$$\Omega(6, K^*) \approx 330. \quad (63)$$

Recalling that the width of the diagram is π , we see that the most probable shape for the Penrose diagram in the pre-inflationary phase is a rectangle about 100 times as high as it is wide. This is ample to allow for chaotic mixing during that phase.

Solving equation (39) in this case, we have $X \approx 14.05$. Thus during the pre-inflationary era the torus grows from an initial side length $2\pi K^* \approx 0.023L_{\text{inf}}$ — *that is, roughly the string scale* — to $2\pi K^* \cosh^{(1/3)}(14.05) \approx 2L_{\text{inf}}$; that is, Inflation begins at about the time global causal contact is lost, as we hoped.

We have given evidence that the FST wave function does naturally predict Linde's [18] scenario. In detail, assuming that the Universe tunnels from “nothing” in the form of a torus — other flat three-manifolds with more complicated topologies would give similar results — we find that the *most probable* initial side length is about the size of the string scale, in agreement with string gas cosmology [17]. The torus then expands through a

pre-inflationary phase, during which all parts of the Universe remain in causal contact. The spacetime geometry during this phase, granted the many simplifications we have assumed, is *most probably* described by a metric of the approximate form

$$g(6, \frac{L_{\text{inf}}}{270})_{+---} = dt^2 - \left(\frac{L_{\text{inf}}}{270}\right)^2 \cosh^{(2/3)}\left(\frac{3t}{L_{\text{inf}}}\right) [d\theta_1^2 + d\theta_2^2 + d\theta_3^2]. \quad (64)$$

This phase ends roughly when global causal contact is lost, by which time the torus is of about the size of the usual inflationary length scale. At that time, the appropriate conditions for Inflation to begin are satisfied, and then the evolution proceeds as usual.

9. Conclusion

Motivated by string gas cosmology [17], we have proposed a model in which the Universe is created, from “nothing”, in the form of a torus. This, through the FST wave function, allows a self-consistent account of the beginning of Inflation.

Naturally we do not claim that the model presented here is realistic, and indeed much would need to be done to make it so. One would need to work with a more precise version of the FST wave function, to include the effects of conventional matter, to account for the details of the end of the NRC-violating effects [“crossing the Phantom divide”, in Hu’s [62] terminology; see [31][63]], to give a more detailed description of the NRC-violating effects and use it to derive a more realistic metric, and so on. Nevertheless there are many aspects of our discussion which can be expected to survive in a more realistic formulation. For example, the inclusion of conventional matter can be expected, in accordance with the Gao-Wald theorem [27], to help to make the pre-inflationary Penrose diagram somewhat *taller*; so we can expect that our prediction — of pre-inflationary Penrose diagrams that are much taller than they are wide — is robust. Again, the Andersson-Galloway results are independent of our special simplifying assumptions, so it is clear that NRC violation in the very early Universe is generic if we assume that the Universe tunnelled from “nothing” to a torus. But NRC violation will be described by some generalized version of the parameter γ discussed in this paper, and these new parameters will have to be fixed in some way. Our discussion here suggests that this will be accomplished by means of a combination of string dynamics and probabilistic arguments based on the wave function itself. These are the general, and most important, lessons of this simplified investigation.

We have noted that controversy continues as to the real status of the Hartle-Hawking and FST wave functions [56]. In particular, Linde objects that these wave functions should not be interpreted, as for example by Firouzjahi et al [7] and Ooguri et al [9], in terms of “probabilities of creation” at all; indeed, in [18] he puts forward the intriguing suggestion that they have to do with *final* rather than with initial conditions. Linde’s basic objection is that, physically, it is not reasonable that it should be easier to create a larger universe than a smaller one. Now the HH wave function suggests that the Universe should be born arbitrarily large; the FST wave function, applied to cosmologies with locally spherical sections, suggests that it should be born at about the inflationary scale; and we have argued that, applied to cosmologies with compact flat sections, the preferred length scale is still smaller, about the string scale. In view of this tendency towards smaller scales, it is natural to ask whether some further refinement might indeed bring the predicted

length scale down to the Planck scale, as Linde prefers. This may be possible if the FST parameter c , which is determined by details of decoherence physics [12], can be made very much larger [see equation (60)] than the value assumed here and in [7]; one may also need to assume a smaller value for the string length scale. These may not seem very plausible assumptions, but one should bear in mind that the “matter” being considered here is very unusual. Clearly this point merits further investigation.

Our combination of Linde’s proposal with that of FST pushes the tunnelling event to [or maybe even below] the string length scale, and so it brings to the fore, once again, the difficult question of how cosmology can be reconciled with the basic machinery of string theory: in particular, with the existence of an S-matrix. Following Bousso [64], we wish to stress that, in cosmological applications, non-compact spatial sections are not particularly helpful if we wish to take an S-matrix point of view. In fact, they can make the puzzle worse. Consider for example the decelerating FRW models with *non-compact* flat spatial sections. These have a spacelike Big Bang singularity, and an observer at any finite time can be aware of only an “infinitely negligible” fraction of the information stored on a spatial slice. In this situation, apart from any other difficulties, one has to deal with the fact that the initial state is almost entirely unknown to any given observer. By contrast, if the spatial sections are tori, *and if the geometry is such that an entire section is visible at a finite time* during the pre-inflationary era, then a given “observer” can be fully informed of the state of the entire Universe by the time Inflation starts.

It seems then that the picture of a globally causally connected pre-inflationary era may throw some light on Bousso’s concerns. But this will only work if K is sufficiently small and if the pre-inflationary geometry has the right form. Thus we have another strong motive to investigate the possibility that the FST wave function might naturally favour values of K which are small relative to the spacetime curvature scale, and spacetime geometries which allow global causal contact.

Other reasons for thinking that compact flat spatial sections may be directly relevant to string cosmology are given in [65]: the existence or non-existence of winding modes for tachyons means that the physics in the case of toral spatial sections is very different from the spherical case. This may in turn prove relevant to our concerns, since [65] indicates that there may be a specifically stringy version of creating the Universe from “nothing”.

We have avoided any discussion of the difficult questions surrounding the state of the Universe in its earliest moments [20][21]. We believe that these questions can only be answered by string theory, perhaps in the guise of string gas cosmology [66]. It is natural to hope that the FST ideas will be helpful here, and it is a good sign that they are compatible with the string gas approach.

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